Viral marketing
with Stochastic optimal control of TPP

HUMAN-CENTERED MACHINE LEARNING
http://courses.mpi-sws.org/hcml-ws18/
Maximizing activity in a social network

Can we steer users’ behavior to maximize activity in a social network?
Endogenous and exogenous events

**Exogenous activity**
Users’ actions due to drives external to the network

**Endogenous activity**
Users’ responses to other users’ actions in the network
Multidimensional Hawkes process

For each user $u$, actions as a counting process $N_u(t)$

Intensities or rates (Actions per time unit)

$$\lambda^*(t) = \mu_0 + A \int_0^t \kappa(t - s) dN(s)$$

User influence matrix

Non-negative kernel (memory)

Exogenous actions

Endogenous actions
Steering endogenous actions

\[ \lambda^*(t) = \mu_0 + A \int_0^t \kappa(t-s) dN(s) + A \int_0^t \kappa(t-s) dM(s) \]

Intensities of directly incentivized actions

\[ \mathbb{E}[dM(t)|\mathcal{H}(t)] = u(t) dt \]

[Zarezade et al., 2018]
Cost to go & Bellman’s principle of optimality

Optimization problem

\[
\begin{aligned}
\text{minimize} & \quad \mathbb{E}(N,M)(t_0,t_f) \left[ \phi(\lambda(t_f)) + \int_{t_0}^{t_f} \ell(\lambda(t), u(t)) \, dt \right] \\
\text{subject to} & \quad u_i(t) \geq 0, \quad \forall t \in (t_0, t_f], \quad i = 1, \ldots, n
\end{aligned}
\]

Dynamics defined by Jump SDEs

\[
d\lambda(t) = [w\mu_0 - w\lambda(t)] \, dt + A \, dN(t) + A \, dM(t)
\]

To solve the problem, we first define the corresponding optimal cost-to-go:

\[
J(\lambda(t), t) = \min_{u(t,t_f)} \mathbb{E}(N,M)(t,t_f) \left[ \phi(\lambda(t_f)) + \int_t^{t_f} \ell(\lambda(s), u(s)) \, ds \right]
\]

The cost-to-go, evaluated at \( t_0 \), recovers the optimization problem!

[Zarezade et al., 2018]
Cost to go & Bellman’s principle of optimality

This is a stochastic optimal control problem for jump SDEs (we know how to solve this!)

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\[
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\]

The cost-to-go, evaluated at \(t_0\), recovers the optimization problem!

[Zarezade et al., 2018]
Lemma. The optimal cost-to-go satisfies Bellman’s Principle of Optimality

\[ J(\lambda(t), t) = \min_{u(t,t+dt)} \left\{ \mathbb{E}_{(N,M)}(t,t+dt) \left[ J(\lambda(t+dt), t+dt) \right] + \ell(\lambda(t), u(t)) \right\} dt \]

\[ dJ(\lambda(t), t) = J(\lambda(t+dt), t+dt) - J(\lambda(t), t) \]

\[ 0 = \min_{u(t,t+dt)} \left\{ \mathbb{E}_{(N,M)}(t,t+dt) \left[ dJ(\lambda(t), t) \right] + \ell(\lambda(t), u(t)) \right\} dt \]

\[ d\lambda(t) = [w\mu_0 - w\lambda(t)] dt + A dN(t) + A dM(t) \]

Hamilton-Jacobi-Bellman (HJB) equation

\[ \text{Partial differential equation in } J \]

(\text{with respect to } \lambda \text{ and } t) \quad [\text{Zarezade et al., 2018}]
Solving the HJB equation

Consider a quadratic loss

\[
\ell(\lambda(t), u(t)) = -\frac{1}{2} \lambda^T(t) Q \lambda(t) + \frac{1}{2} u^T(t) S u(t)
\]

Rewards organic actions

Penalizes directly incentivizes actions

We propose \( J(\lambda(t), t) \) and then show that the optimal intensity is:

\[
u^*(t) = -S^{-1} \left[ A^T g(t) + A^T H(t) \lambda(t) + \frac{1}{2} \text{diag}(A^T H(t) A) \right]
\]

Computed offline once!

Closed form solution to a first order ODE

Solution to a matrix Riccati differential equation

\[\text{[Zarezade et al., 2018]}\]
The Cheshire algorithm

Intuition
Steering actions means sampling action user & times from $u^*(t)$

More in detail
Since the intensity function $u^*(t)$ is stochastic, we sample from it using:

- Superposition principle
- Standard thinning

It only requires sampling $1^T N(t_f)$ from inhomog. Poisson!

[Zarezade et al., 2018]
Experiments on real data

Five Twitter datasets (users) where actions are tweets and retweets

1. Fit model parameters

\[ d\lambda(t) = [w\mu_0 - w\lambda(t)] \, dt + A \, dN(t) \]

↑ exogeneous rate  ↓ influence matrix

2. Simulate steering endogenous actions

\[ d\lambda(t) = [w\mu_0 - w\lambda(t)] \, dt + A \, dN(t) + A \, dM(t) \]

↑ directly incentivized tweets chosen by each method

[Zarezade et al., 2018]
Evaluation metrics & baselines

Evaluation metrics

- \( \bar{N}(t) = \sum_{u \in \mathcal{V}} \mathbb{E}[N_u(t)] \)
  - Average number of not directly incentivized tweets
- \( \bar{t}_{30K} \)
  - Average time to reach 30,000 not directly incentivized tweets

Baselines

- MSC [Farajtabar et al., NIPS ’16]
- OPL [Farajtabar et al., NIPS ’14]
- PRK (Pagerank)
- DEG (Out-degree)

[Zarezade et al., 2018]
Performance vs. time

Cheshire (in red) triggers 100%-400% more posts than the second best performer.

[Zarezade et al., 2018]
Performance vs. # of incentivized tweets

Cheshire (in red) reaches 30K tweets 20-50% faster than the second best performer

Series, $M(t_f) \approx 5k$

[Zarezade et al., 2018]
Why Cheshire?

“the Cheshire Cat has the ability to appear and disappear in any location”

Alice’s Adventures in Wonderland, Lewis Carroll